Statistical modeling of extreme value behavior in paleoclimate proxies

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Introduction

- Growing interest in studying the changing behavior of climatic extremes in both space and time.
  - The most acute societal impacts of climate change may be those arising from changes in the frequency and severity of extreme climate events (Zwiers and Kharin 1998; IPCC 2007, Chapter 3).
- The statistical theory of extreme values is a well-developed formalism for investigating extremal characteristics [e.g. Coles, 2001].
  - Increasingly being applied to the analysis of the instrumental climate record and climate model output.

Extreme value parameter modeling

- A common strategy for analyzing climate-model-derived temperature or precipitation fields is to treat the **parameters** of the extreme value distribution as temporally or spatially varying quantities [Kharin and Zwiers, 2000, 2005, Schliep et al., 2010, Mannshardt-Shamseldin et al., 2010].

- Katz [2010, p.5] argues shifts in the frequency and magnitude of climate extremes can be reliably derived by modeling the temporal and spatial behaviour of the probability distribution characterizing the climate variable, rather than the extremes themselves.

- This points to the importance of understanding and modeling of extremes.

Where do proxies enter the stage?

- To place recently observed extremes in the climate system into a **longer term** context than is possible using the instrumental record, it is necessary to turn to the climate proxy record.
- There have been numerous efforts to infer both the **spatial mean** and **spatial pattern** of past surface temperatures from natural proxies.
  - Reviews: NRC [2006], IPCC [2007, Chapter 6], and Jones et al. [2009].
- Usually climate is inferred from proxies using **regression-type** approaches.
  - Provides important insights into long-term climate behavior.
  - But limited in being able to answer questions about the distribution of extremes [see, e.g., NRC, 2006, Field et al., 2012].

Extreme value theory in climate proxy data

- It is more appropriate to use extreme value theory (EVT) to directly model the tails of a distribution:
  - Katz et al. [2005] model sediment yield series using a generalized extreme value (GEV) distribution. They highlight that modeling spatio-temporal effects is key in ecology; their model only includes temporal components.
  - Naveau and Ammann [2005] develop a time series of extreme events based on ice core sulfate proxies (not modeling distribution of extremes).
  - Cooley et al. [2006] model the age of moraines, using a GEV distribution for lichen measurements. With a separable spatio-temporal model to model the location parameter, spatial effects are modeled via a random effect term; the shape and scale parameters are held fixed.

Our aim

- To investigates both the **spatial and temporal patterns** in the extremal characteristics of climate proxies by applying EVT to a set of tree ring density series over northern North America.

- Model can **predict** extreme value behavior at unobserved locations.

- To the extent that extremes in tree ring densities reflect extremes in climate, any significant temporal and spatial variations detected in the extremal behavior of the tree ring densities are indicative of changes in the extremal characteristics of the climate system.

- Can they extend to reconstruction of extremal behavior of the climate variables for which the proxies are informative.

Block modeling approaches

- We use a generalized extreme value (GEV) distribution approach for block maxima (and minima).

- At any individual location $\boldsymbol{s}$, suppose we observe the time series

$$\{Y_t(\boldsymbol{s}) : t = 1, \ldots, T(\boldsymbol{s})\},$$

of a length $T(\boldsymbol{s})$ that is divisible by a block length B.

- Calculate the maxima for each block:

$$M_j(\boldsymbol{s}) = \max\{Y_{(j-1)B+1}(\boldsymbol{s}), \ldots, Y_{jB}(\boldsymbol{s})\}.$$

(e.g., decadal maxima)

The GEV distribution

- Under certain regularity conditions, the series of block maxima asymptotically follow a **generalized extreme value** (GEV) distribution [e.g., Coles, 2001], which we denote by $M_j(\boldsymbol{s}) \sim \mathrm{GEV}(\eta, \sigma, \xi)$.

- Assuming that the parameters of the distribution are independent of time and space, the cumulative distribution function is

$$\Pr\{M_j(\boldsymbol{s}) \leq x\} \;=\; \exp\left\{-\left(1 + \xi\left[\frac{x-\eta}{\sigma}\right]\right)_+^{-1/\xi}\right\},$$

where $y_+ = \max\{y, 0\}$.

- The GEV approximation improves for longer block lengths, B.

The GEV parameters

- $\eta \in \mathbb{R}$ is the **location parameter**, with a larger value concentrating the distribution of the maxima at higher values.
- The **scale parameter**, $\sigma > 0$, determines the spread of the distribution, with higher values resulting in a more disperse distribution of the maxima.
- The **shape parameter**, $\xi \in \mathbb{R}$, describes the tail behavior of the distribution, with higher values corresponding to heavier tails. When the shape parameter is negative, the tails are bounded; otherwise the tails are unbounded.

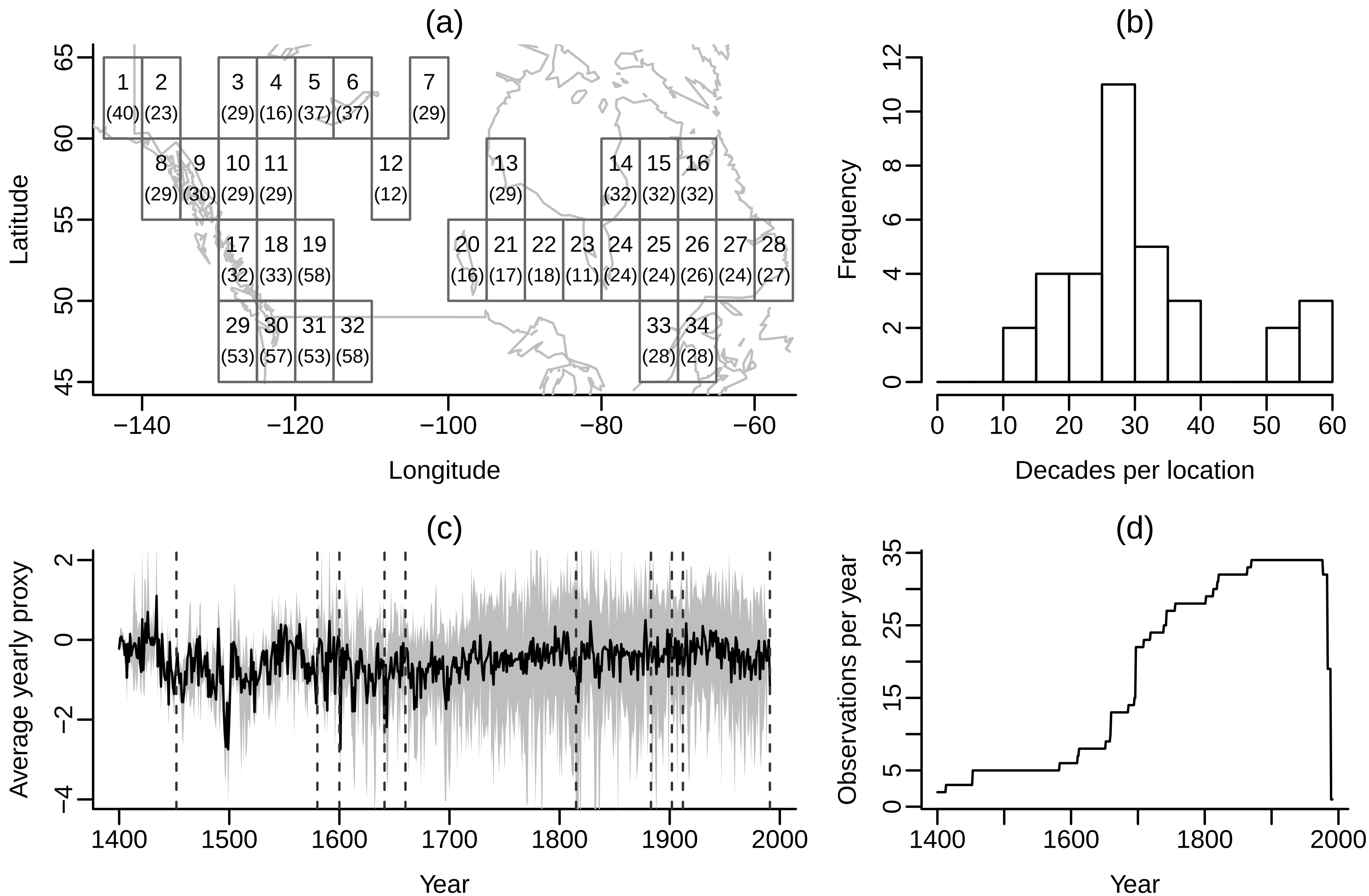
The choice of extreme value modeling

- Other choices:
  - **The running-maxima approach** (generates more maximum values to study, but introduces dependence between the maxima)
  - **The points (or peaks)-over-threshold approach**, leading to the *generalized pareto distribution* (GPD) [Pickands, 1975] (not enough data values for our proxy).
- All these methods lead to a **marginal extreme value analysis**

  (We leave a multivariate analysis for further research).

The proxies: annually resolved tree ring densities

- We use a gridded version of the maximum late wood density data set described in Briffa et al. [2002a,b].
  - Used in numerous efforts to reconstruct past climate.
  - Grid box values are weighted averages of site-specific chronologies in each box; weights determined by the number of trees in each chronology.
  - Each chronology is formed by averaging cores from individual trees (roughly 20) for a given site, after removing growth effects.
  - Considered to be standardized residuals with arbitrary units.
- Gridded so we can compare to the Climate Research Unit's gridded instrumental temperature data set [Brohan et al., 2006] in the future.


(a)
Latitude
Longitude
1 (40)
2 (23)
3 (29)
4 (16)
5 (37)
6 (37)
7 (29)
8 (29)
9 (30)
10 (29)
11 (29)
12 (12)
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14 (32)
15 (32)
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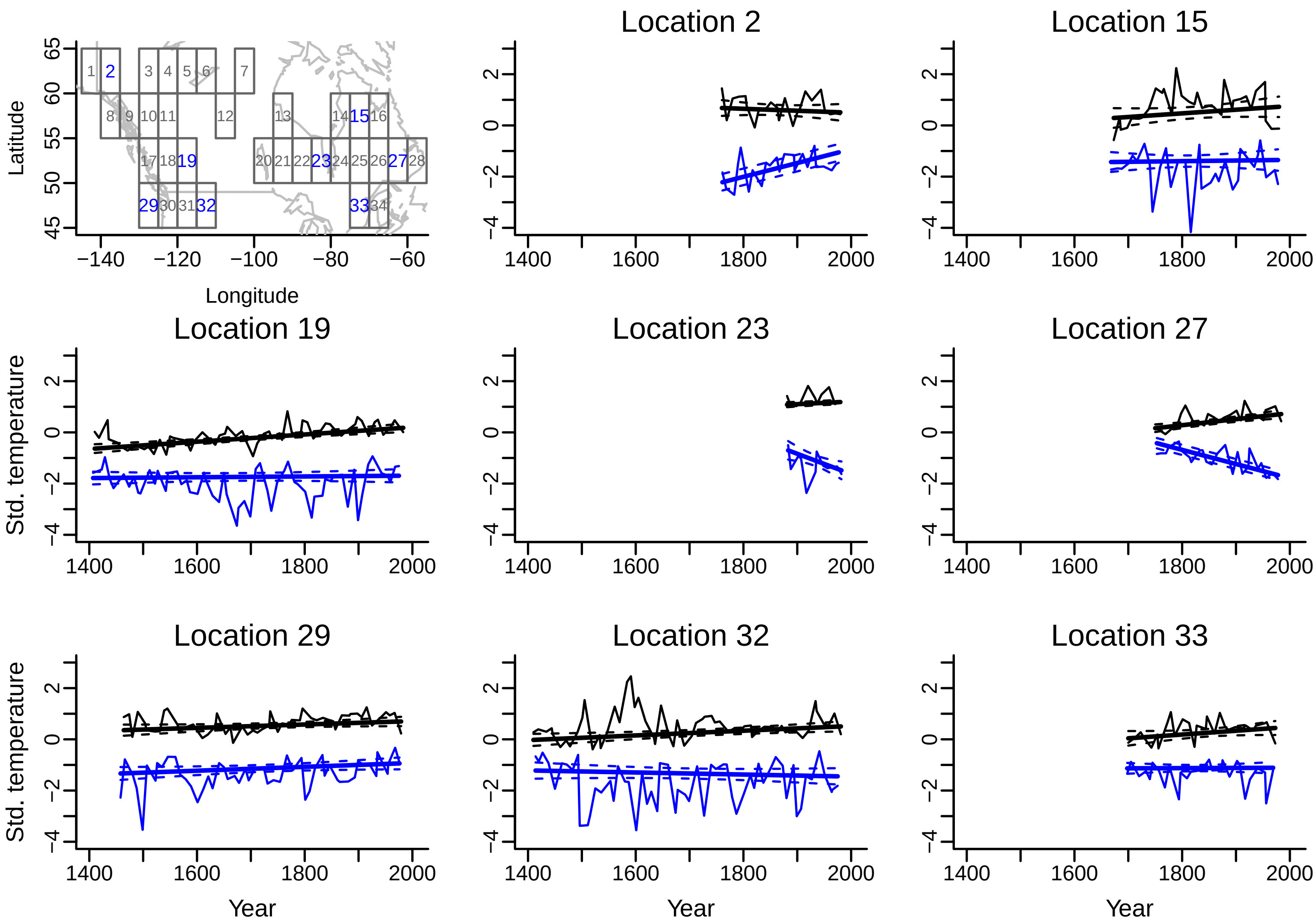
17 (32)
18 (33)
19 (58)
20 (16)
21 (17)
22 (18)
23 (11)
24 (24)
25 (24)
26 (26)
27 (24)
28 (27)
29 (53)
30 (57)
31 (53)
32 (58)
33 (28)
34 (28)
(b)
Frequency
Decades per location
(c)
Average yearly proxy
Year
(d)
Observations per year
Year

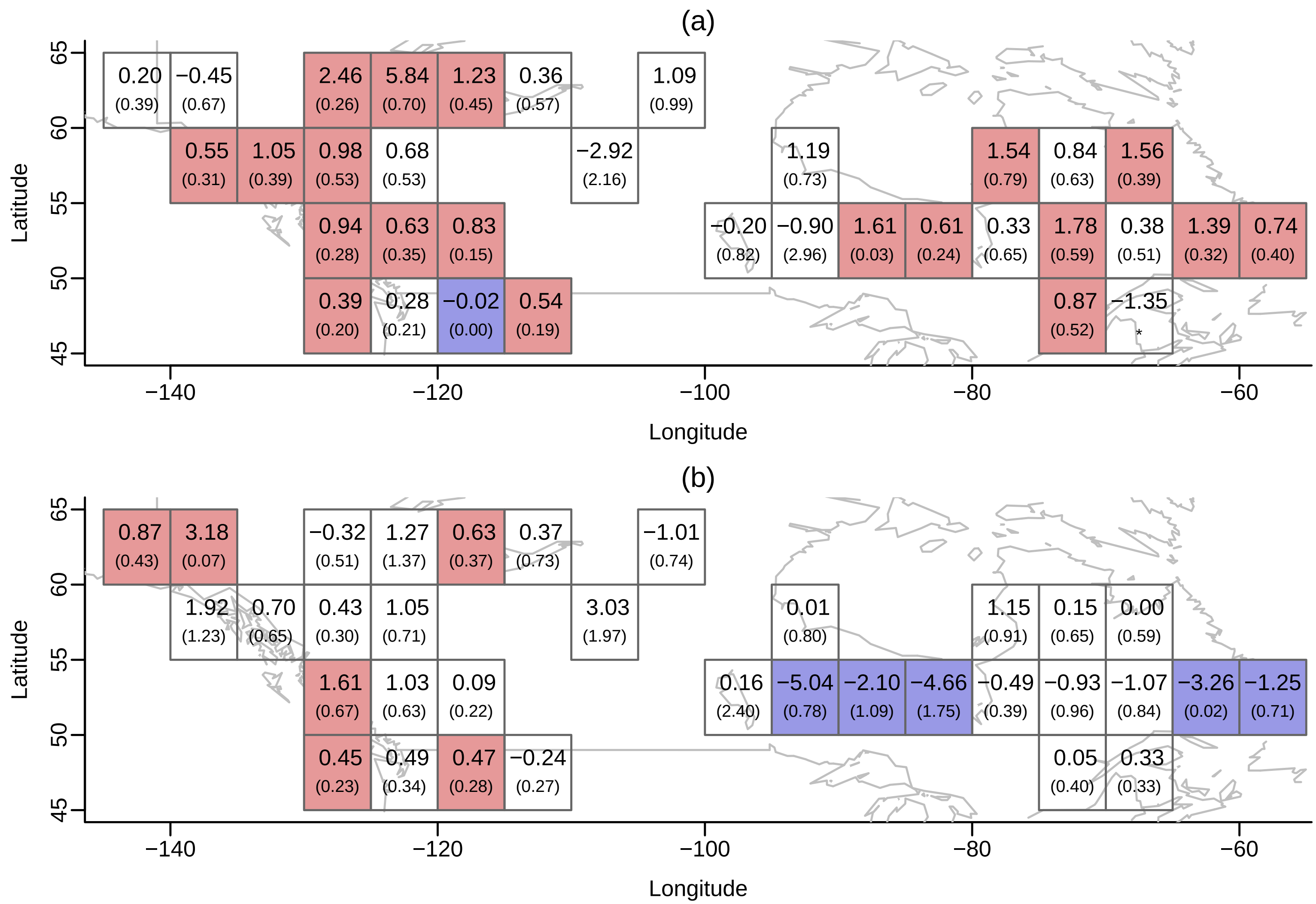
Exploratory extreme value analysis

- Derive **decadal maxima** and **minima** tree ring series at each grid box.
- There is clear evidence of spatially varying temporal dependencies.
- At the majority of locations, the decadal maxima exhibit increasing long-term trends of differing magnitudes.
- The decadal minima are more mixed: west exhibits increasing trends (e.g., 2 and 29); some areas in east are decreasing (e.g., 23 and 27).
- For the most part, the variability of the decadal minima (with respect to the temporal trend) appears larger than that for the the decadal maxima.
- We also carried out some site-by-site GEV analyses.

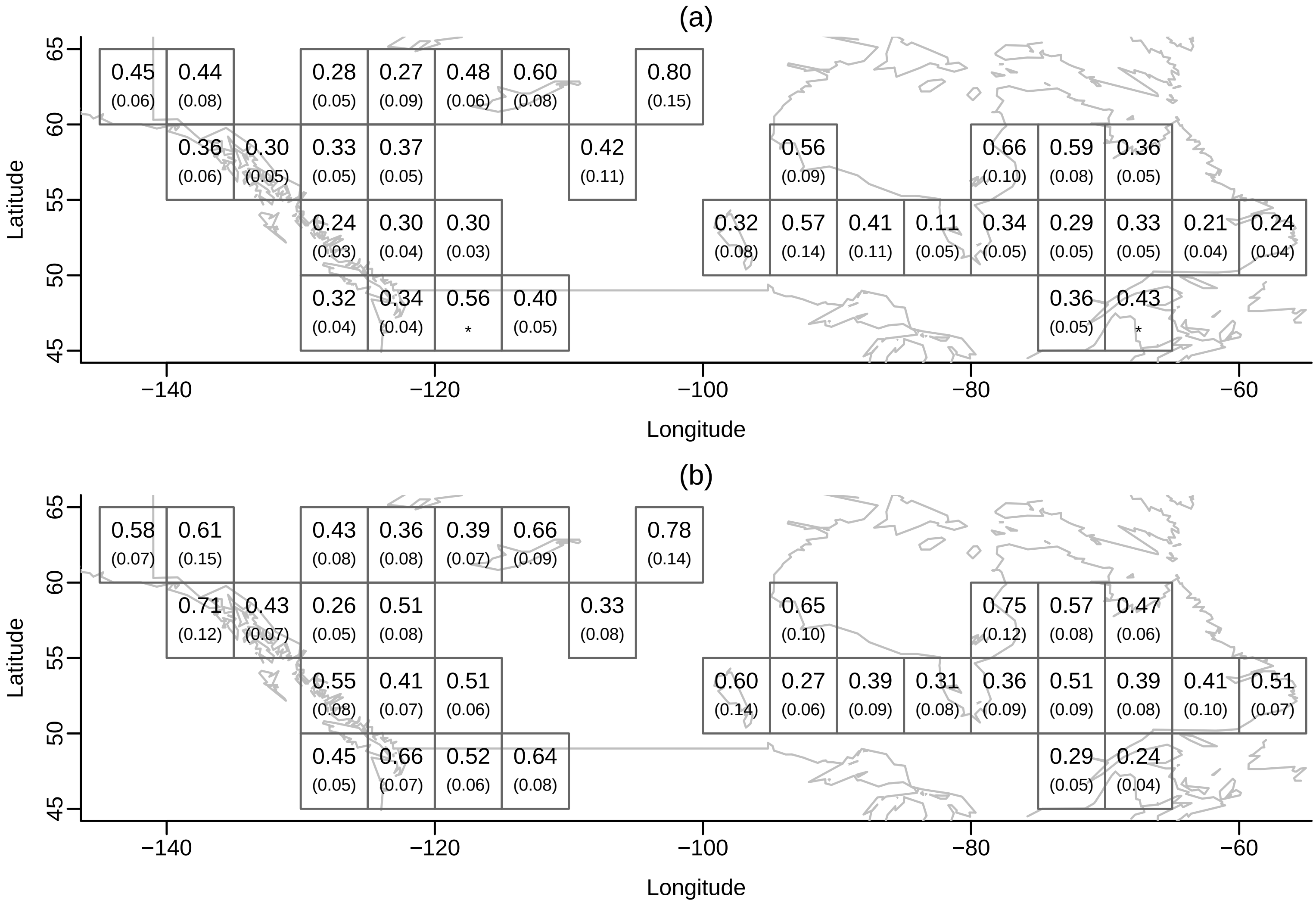


Location 2
Location 15
Location 19
Location 23
Location 27
Location 29
Location 32
Location 33
Latitude
Longitude
Std. temperature
Year

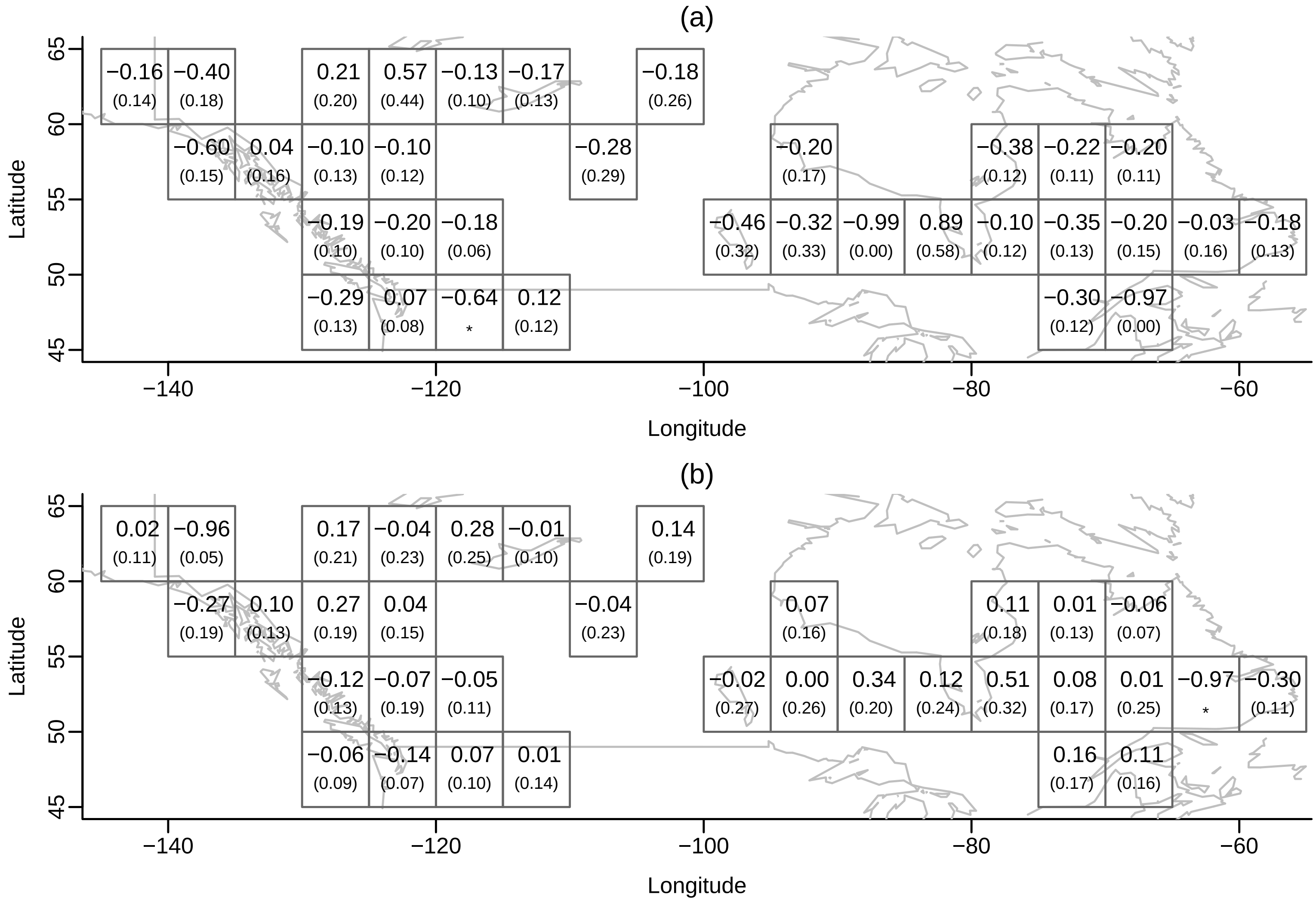
EDA: site-by-site slope estimates



EDA: site-by-site scale estimates



EDA: site-by-site shape estimates



Bayesian hierarchical modeling of paleoclimate extremes

- We assume that the distribution of the maxima (or minima) over decades $j = 1, \ldots, N(\boldsymbol{s}_i)$ and locations $\boldsymbol{s}_i$ ($i = 1, \ldots, K$) are independent, conditional on the parameters of the GEV distribution, with

$$M_j(\boldsymbol{s}_i)|\eta_j(\boldsymbol{s}_i), \sigma, \xi \;\sim\; \mathrm{GEV}(\eta_j(\boldsymbol{s}_i), \sigma, \xi). \tag{1}$$

- The location parameter $\eta_j(\boldsymbol{s}_i)$ satisfies

$$\eta_j(\boldsymbol{s}_i) \;=\; \alpha(\boldsymbol{s}_i) + \beta(\boldsymbol{s}_i) a_j(\boldsymbol{s}_i),$$

where $a_j(\boldsymbol{s}_i) = (\mathrm{yr}_j(\boldsymbol{s}_i) - 1405)/600$ is a standardized year variable.

- The scale and shape parameters (σ and ξ) are modeled as each being constant over space and time, a modeling choice motivated by EDA.

Spatially varying intercepts and slopes

- The **spatially varying** coefficient models for the intercepts, $\alpha(\boldsymbol{s})$, and slopes, $\beta(\boldsymbol{s})$, are specified in terms of Gaussian processes.
- The process means are linear in the longitude and latitude.
- The stationary, spatial process covariance structures captures residual spatial dependence between locations, while allowing for a nugget effect that captures possible site-by-site heterogeneity.
- We use a stationary exponential correlation, but the chordal distance metric.
  - Induces "a valid correlation function on the sphere" [Banerjee, 2005], and allows for the interpretation of the range parameters in units of kms.

Bayes inference

- Where possible, **conjugate** prior distributions were employed to simplify the inference.
  - But, we used information from other studies about **correlation lengths** to set specify priors for the spatial range parameters.
- With $\boldsymbol{\eta} = (\eta_j(\boldsymbol{s}_i))$ denoting the collection of location parameters, the parameter vector is

$$\boldsymbol{\theta} = (\boldsymbol{\eta}, \sigma, \xi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}_\alpha, \tau_\alpha^2, \phi_\alpha, \omega_\alpha^2, \boldsymbol{\lambda}_\beta, \tau_\beta^2, \phi_\beta, \omega_\beta^2).$$

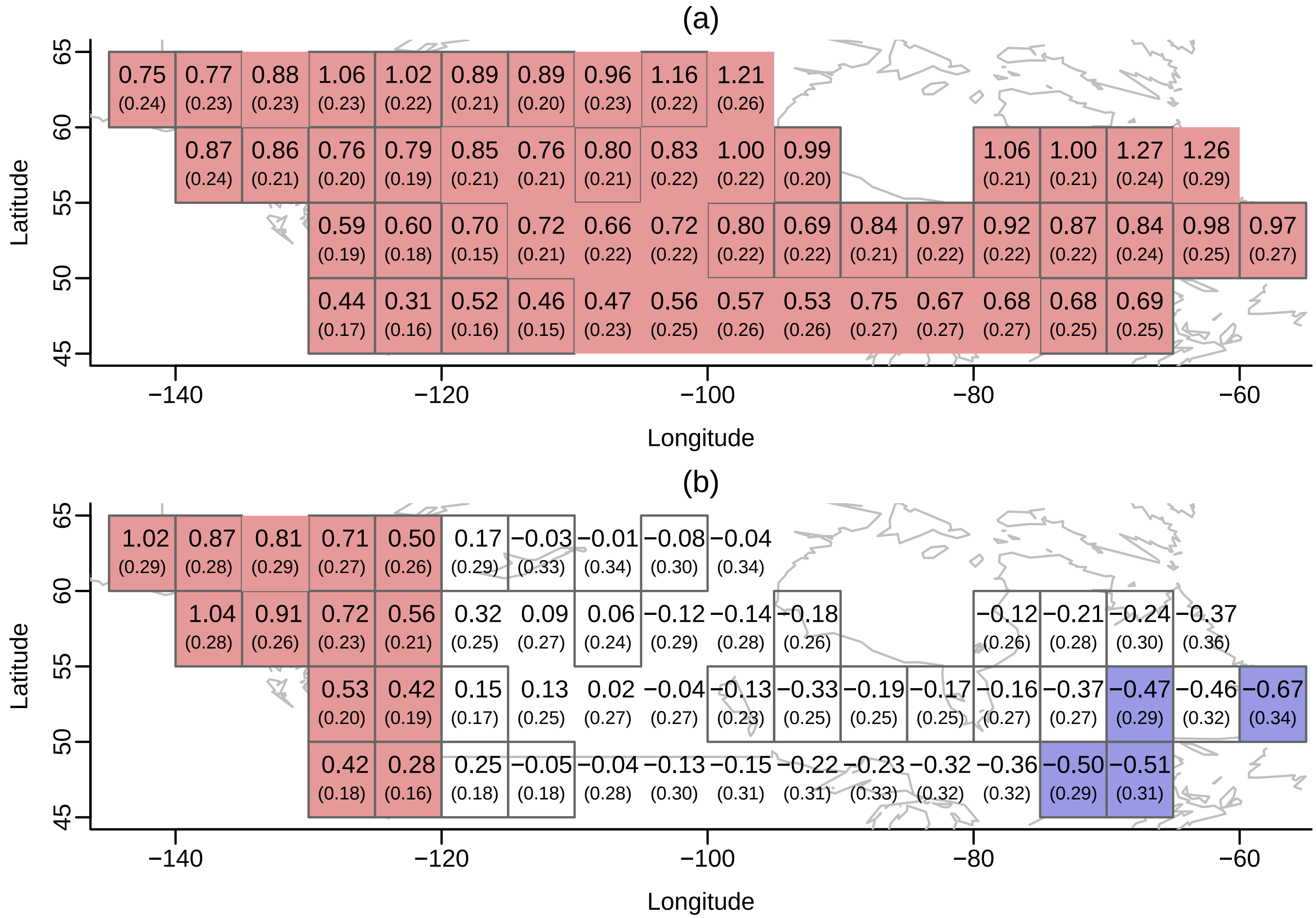
Markov chain Monte Calro

- The posterior does not follow a well-known closed form, so we use a Markov chain Monte Carlo (MCMC) algorithm.
  - The first 10,000 samples were discarded as a "burn-in".
  - Next 100,000 (thinned every 20) we used for inference.
  - We assess convergence using multiple chains and trace plots.

Results

Parameter	Decadal Maxima Post. Mean	Decadal Maxima Post. 95% CI	Decadal Minima Post. Mean	Decadal Minima Post. 95% CI
(scale) σ	0.414	(0.395, 0.434)	0.546	(0.518, 0.574)
(shape) ξ	-0.085	**(-0.120, -0.047)**	0.000	(-0.039, 0.043)
(intercept) $\lambda_{\alpha,1}$	0.525	(-2.044, 3.106)	2.626	**(0.413, 4.834)**
(long) $\lambda_{\alpha,2}$	-0.007	(-0.021, 0.006)	0.010	(-0.002, 0.020)
(lat) $\lambda_{\alpha,3}$	-0.024	(-0.070, 0.019)	-0.056	**(-0.094, -0.019)**
(spatial var) τ_α^2	0.151	(0.043, 0.331)	0.080	(0.013, 0.202)
(range par) ϕ_α	1083.719	(573.474, 1767.003)	1098.144	(574.274, 1782.636)
(nugget) ω_α^2	0.023	(0.004, 0.073)	0.015	(0.003, 0.048)
$\lambda_{\beta,1}$	-0.561	(-2.528, 1.489)	-2.323	(-4.909, 0.493)
$\lambda_{\beta,2}$	0.006	(-0.003, 0.015)	-0.017	**(-0.031, -0.005)**
$\lambda_{\beta,3}$	0.036	**(0.002, 0.071)**	0.012	(-0.036, 0.055)
τ_β^2	0.038	(0.004, 0.152)	0.078	(0.007, 0.246)
ϕ_β	1020.224	(488.242, 1757.427)	1095.691	(565.204, 1806.958)
ω_β^2	0.022	(0.004, 0.069)	0.017	(0.003, 0.057)

Results, continued



Model assessment

- Quantile-quantile plots were used to assess the distributional assumptions made by the GEV models [see, e.g., Coles, 2001].
  - The GEV models fit better for the decadal maxima than for the minima.
- To investigate the robustness of our conclusions to changes in the modeling and prior assumptions, we considered additional models for the decadal maxima (and minima) of the tree ring density series.
  - We obtain qualitatively similar results and conclusions.

Conclusions

- While many reconstructions of late Holocene climate include statements about the extent to which recently observed climate is, by some metric, extreme [e.g., Mann et al., 1999, Luterbacher et al., 2004, Kaufman et al., 2009, Barriopedro et al., 2011], such studies do not exploit the power of EVT.

- Application of EVT to a suite of climate sensitive [Mann et al., 2008, Briffa et al., 2002a,b] late wood tree ring density series over Northern North America reveals a rich spatio-temporal structure in the distributional parameters governing the extremal behavior of the proxies.

Conclusions, continued

- Decadal maxima are trending upwards over this spatial region.
- Decadal minima: trending up in the west, some downward trends in east.
- Taken together, the range in tree ring densities is increasing faster in the east than the west, and the distribution is more volatile in the east.
- Consistent with Field et al. [Fig. SPM.3 from the IPCC SREX; 2012]. (Managing the Risks of Extreme Events)
  - To the extent that the extremes in the proxies reflect extremes in the climate, specifically temperature, theses significant temporal and spatial variations in the extremal behavior of the proxies are indicative of similar changes in the extremal characteristics of the climate system.

Extensions

- Build richer models (especially including covariates such as green house gas concentrations, solar irradiance, and volcanic forcing [c.f., Li et al., 2010]).
- Applications to other proxies.
- Linking climate and proxies.

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